Automatic registration for 3D shapes using hybrid dimensionality-reduction shape descriptions

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1. Introduction

Over the past decades, commercially available laser range sensors have been the popular tools for accurately measuring the 3D objects in computer vision. One basic problem usually encountered is the shape registration, finding the rigid transformation parameters between two images and aligning them as closely as possible. Many practical applications benefit from the registration, such as reverse engineering, motion estimation, face recognition, rapid prototyping and even robot navigation. However, it is often difficult to obtain a satisfactory registration in the presence of different resolution, noise, occlusion and non-overlapping data. Nowadays, automatic registration for 3D shapes has become a challenging task with both theoretical interest and practical importance.

1.1. Previous work

Many methods have been proposed to tackle the problem. A popular approach is to use salient features as invariant to rigid transformation, in order to search for correspondences with similar features. For instance, Krsek et al. [1] defined level curves of constant mean curvature as the invariant. Then, correspondences search was performed overall potential points in scans. Typical local features used for registration include line features [2,3], volumetric features [4,5] and multi-scale features [6]. These methods are simple, fast and do not require the initial transformation. The main drawback is that they may fail when the scans contain few local salient features.

Iterative closest point (ICP) [7] and its improved algorithms [8,9] are the famous registration methods without feature extraction. A landmark contribution for ICP-based algorithms is to use the iterative procedure to obtain the local optimal value. At each iteration, a distance function between points and their correspondences is used to compute the rigid transformation. However, they require a good initial estimate of motion parameters, and when there are a large number of points in scans, seeking correspondences, which involves exhaustive inquiry of closest points is quite time-consuming. More recently, genetic algorithms [10,11] have been investigated to solve the global solution to transformation, in terms of chromosomal variation. They dispense with an initial transformation and work well in the presence of noise or outliers. However, it is usually time-consuming for convergence.

There is a big literature on the different varieties of surface representation methods, which have been applied to handle the registration. Spin-Image [12] is one of the most famous surface representation-based registration methods. Consider a given point on surface, 2D image is generated from defined tangent plane and normal distance. By correlation of images, points on sensed surface and their correspondences on model surface can be found. Then, geometric consistency is used to remove false
correspondences, and the transformation parameters can be computed. The main advantage of this method is that it is simple to implement. However, the calculation efficiency is related to point matches. When there are many regionally similar surface patches, the computation time is high. Based on the correlation of two Extended Gaussian Images (EGIs) in the Fourier domain, Makadia et al. [13] presented a registration method without manual initial registration or artificial landmarks. Crude alignment

Select seeds on sensed surface

Select vertices on model surface

Construct local DRSDs

Construct local DRSDs to establish a library

Compute local normalized correlation coefficient to generate the candidates

Apply the distance root mean squared error to remove the outliers in candidates

Use a updated group of triplets to calculate the rigid transformation

Refine transformation parameters by a variational ICP algorithm

Calculate the rigid transformation

Optimal transformation parameters

Construct global DRSD with respect to sensed surface

Select a fixed triplet

Construct global DRSD with respect to model surface

Build a plane chessboard to verify overlapping regions

Compute global normalized correlation coefficient for registration error

Evaluate the registration accuracy

Fig. 1. Block diagram of hybrid DRSD algorithm.
is achieved by the constellation images from EGlS. Then, ICP algorithm is used to refine the rigid transformation parameters. Specifically, this method is suitable for multi-scans shape registration with little overlap.

Other surface representation-based methods include point-signature representation [14,15], symbolic description [16], surface signature representation [17], spherical representation [18,19], algebraic polynomial representation [20], shape fingerprint representation [21], tensor representation [22] and neural network representation [23]. For more details on this topic, we refer the interested readers to [24,25]. Generally, local surface patches are used to construct invariant where the intrinsic shape information of surface is used for correspondences. The registration using surface representation does not require initial estimation of the transformation. In addition, only few point matches are required to solve the best transformation, not like iterative methods that require many points to participate in the calculation. The common drawback is that they have difficulty in defining discriminate similarity metric to determine the possible correspondences. Moreover, when there are partially overlapping regions, it still remains a challenging task about how to compute the registration error accurately and effectively, meanwhile avoiding the search of closest points in 3D space as much as possible.

1.2. Surface representation using mesh parameterization

Recently, surface representation methods based on mesh parameterization have been studied extensively in various computer graphics and geometry processing applications, such as texture mapping, remeshing and even toolpath generation in CNC machining. Mesh parameterization can be viewed as an embedding from a 3D surface with similar topology to a 2D domain, involving a bijective map between a piecewise mesh surface and a suitable planar domain. Based on the shape-preserving properties, Hornmann et al. [26] gave a basic classification of mesh parameterization methods: angle-preserving parameterization, distance-preserving parameterization and area-preserving parameterization.

Due to simple implementation and property of preserving geometric shape, angle-preserving parameterization has attracted much attention in recent years. According to Riemann's theorem, arbitrary mesh surface with similar topology can be mapped conformally onto a planar domain. Hence the 3D geometric processing can be simplified as the problem in 2D plane. Recently, the surface representation methods using angle-preserving parameterization are proposed to handle shape registration and matching. By minimizing an energy function, Zhang and Hebert [27] applied Harmonic Maps with circular boundary to carry out parameterization, in order to seek feature points for surface matching. However, when the meshes contain obtuse angles, there may be “triangle flips” or “winding itself”, which dissatisfies the bijective. Based on some fixed points obtained by Spin-Image, Wang et al. [28] proposed a new shape registration technique using least-squares conformal maps (LSCM). Due to linear parameterization with free boundary, their method was more robust than Zhang’s method with fixed boundary. However, for the regions with high-curvature features, linear parameterization method LSCM will generate large distortion when flattening the overall surface onto plane.

From the contents mentioned above, it can be seen that angle-preserving parameterization with dimensionality reduction is an interesting mathematical tool for surface representation. Inspired by recent work on mesh parameterization, we aim to apply suitable parameterization methods to describe free-form surface. Hence, complex geometric processing can be analyzed and calculated in low-dimensional space.

1.3. Overview of proposed method

In this paper, we propose a hybrid DRSD method to represent free-form surface for automatic registration, based on angle-preserving parameterization techniques such as Harmonic Maps and ABF++. Our registration algorithm consists of two main steps: local DRSD for computing rigid transformation and global DRSD for evaluating registration accuracy. Fig. 1 shows the flowchart of our algorithm. All the individual procedures will be explained in detail in Sections 3 and 4.

The basic idea of hybrid DRSD algorithm is to map 3D surface to a 2D parametric domain. First, local DRSD is constructed to map the selected surface patches to the circular domain in plane using Harmonic Maps. Using the normalized correlation coefficient defined in plane as similarity metric of local DRSD, correspondences are obtained and used for registration. Second, using non-linear parameterization method ABF++, global DRSD is constructed to stretch the overall surfaces onto plane with minimal distortion. Therefore, the overlapping regions of two surfaces are verified and the registration error can be obtained in 2D space. Besides the mathematical intuition, hybrid DRSD algorithm affords other advantages, which are significant to 3D shape registration: (a) they preserve shape and continuity information of underlying surface in low-dimension space, which is evidently different from other surface representation methods; (b) they associate the surface attributes properties in local and global regions as consistent invariant in different spaces, thus making them possible to account for global rigid transformation; (c) the maps can deal with the situation when there is resolution variation, noise and occlusion. All these appealing traits indicate that hybrid DRSD has the potential to simply and efficiently achieve registration.

The rest of the paper is organized as follows. In Section 2, we present the theoretical knowledge of angle-preserving parameterization. In Section 3, we introduce the process of constructing local DRSD, and describe the detailed steps of solving optimal transformation, including seeking correspondence to generate candidates, rejecting the outliers in candidates, selecting triplets to compute rigid transformation and refining the transformation parameters. In Section 4, we present the construction of global DRSD, which is used to verify the overlapping regions for two images and compute the registration error. In Section 5, the time complexity and the limitations are analyzed and discussed. In Section 6, several experiments have been implemented to test the proposed hybrid DRSD algorithm. The conclusions and future work are given in Section 7.

2. Theoretical basis of angle-preserving parameterization

In this section, two methods of angle-preserving parameterization, Harmonic Maps and ABF++, are introduced from the viewpoint of energy minimization. In terms of the geometric properties and calculation complexity, the strengths and weaknesses of the two methods are described, following that hybrid dimensionality-reduction shape descriptions are constructed to perform shape registration in Sections 3 and 4.

2.1. Harmonic Maps

Harmonic Maps are the solutions to partial differential equations, from the Dirichlet energy defined in Riemannian manifolds. As described in [29,30], for 3D surface patches S and planar domain D, Harmonic Maps h:S →D can be obtained by minimizing
the following equation:

$$E_{D}(h) = \frac{1}{2} \int_{S} \left( \|
abla u\|^2 + \|
abla v\|^2 \right)ds = \frac{1}{2} \int_{S} \|
abla H\|^2 ds$$ (1)

where $\nabla u$ and $\nabla v$ denote the local coordinate tensors in smooth Riemann manifolds, $ds$ denotes the area element on $S$ and $E_{D}$ denotes the Dirichlet energy. As for discrete triangles, Eq. (1) can be rewritten as

$$E_{D}(h) = \frac{1}{2} \sum_{(ij \in \text{Edges})} k_{ij} \|h(i) - h(j)\|^2$$

Subject to: $h|_{\partial S} = \partial S \to \partial D$ (2)

where coefficient $k_{ij}$ is the spring constant between point $i$ and point $j$, and $h|_{\partial S}$ is the boundary conditions from $S$ to $D$. By minimizing the Dirichlet energy in Eq. (2), the plane coordinates under Harmonic Maps $h$ are obtained. Harmonic maps, always existing embedded in plane, are one-to-one and intrinsic to 3D shapes, which indicates that they supply a view-independent quantity to rigid transformation. We will use them to construct local DRSD to represent surface patches, searching for the correspondences.

Depending on variations in spring constant and boundary polygon, different Harmonic Maps are obtained. Recently, a common framework of existing spring constants, by Tutte’s uniform coefficient, Eck’s discrete Harmonic coordinates and Floater’s mean-value coordinates, have been described by Hornung et al. [26], mainly for the application of texture mapping. However, to our best knowledge, the differences of applying these spring constants in shape registration are rarely described. Note that Harmonic Maps gives angle-preserving properties only in local regions. However, when overall surfaces are stretched onto 2D plane, they will generate large distortion due to restricted boundary conditions. In this situation, it is hard to keep the consistency of surface geometric properties between 3D physical space and 2D parametric space. This problem can be solved by Angle-Based Flattening (ABF/ABF++) method with free boundary, another mathematical tool for angle-preserving parameterization.

2.2. Angle-Based Flattening

Non-linear parameterization methods ABF/ABF++ were proposed by Sheffer and de Sturler [31] and Sheffer et al. [32]. The objective function $F$ of ABF is defined as follows:

$$F(\alpha_{\text{tri}}, \alpha_{\text{plan}}, \alpha_{\text{len}}) = E + \sum_{i} \alpha_{\text{tri}}(i) + \sum_{v} \alpha_{\text{plan}}(v) + \sum_{v} \alpha_{\text{len}}(v)$$ (3)

The first item $E$ in right part of the equation denotes the energy function defined in angle space, and the last three items denote the constraints in a valid triangle. Using Newton’s method, the non-linear objective function $F$ is minimized by

While $\|\nabla F\| > \varepsilon$

solve $\nabla^2 F \delta = -\nabla F$

$x = x + \delta$

end

To increase the efficiency, ABF was augmented to yield ABF++, which simplifies the direct solver from the Newton to the Gauss–Newton method. At each iteration, the linear system $\nabla^2 F \delta = -\nabla F$ in (4) changes into

$$\begin{bmatrix} A & J \end{bmatrix} \begin{bmatrix} \delta_2 \\ \delta_1 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$ (5)

where

$$J = \begin{bmatrix} \frac{\partial^2 F}{\partial \delta_1 \partial \delta_2} \\ \frac{\partial^2 F}{\partial \delta_1 \partial \delta_1} \\ \frac{\partial^2 F}{\partial \delta_2 \partial \delta_1} \\ \frac{\partial^2 F}{\partial \delta_2 \partial \delta_2} \end{bmatrix}, \quad b_1 = -\nabla_{\delta_2} F, \quad b_2 = -\nabla_{\delta_1} F$$

and $\delta_1$ and $\delta_2$ denote the step vectors.

Then, the dimension of matrix structure is lowered by decomposing the simple Hessian matrix, which dramatically decreases the time consumption of mesh parameterization.

Note that the energy function used in ABF/ABF++ is defined in terms of the angles in triangles. The parameterization method attempts to minimize the deformation of 3D shapes. Sheffer et al. have demonstrated that ABF++ parameterization exhibited significantly less stretch than linear methods, especially for objects with high-curvature features. It provides the theoretical basis to implement dimensionality-reduction strategy for representing overall mesh surfaces. In this paper, applying the superior angle-preserving properties, we expect ABF++ to be the useful tool to construct global DRSD for evaluating the registration accuracy.

3. Compute the optimal transformation by local DRSD

In this section, we will first introduce how to implement the Harmonic Maps to construct local DRSD, in order to solve optimal transformation parameters. Local DRSD is defined to describe surface properties, where the geometric invariant is extracted to seek correspondences. Recently, some local invariant descriptors have been proposed by the researchers, such as Point Signature [14,15], Spin-Image [12,33], RANSAC-based DARCES [34], Surface Signature [17], Harmonic Shape Image [27], etc. In Surface Signature and Harmonic Shape Image, curvature has been explored to construct view-independent shape descriptions for the purpose of shape registration or object recognition. Surface curvature is a useful shape attribute that depends on the surface’s intrinsic geometry. In this paper, surface curvature, which is calculated based on [35], is used to provide invariant features for shape registration.

3.1. Constructing local DRSD

To represent the local surface patches, Harmonic Maps are adopted to construct local DRSD. Fig. 2 shows the process of constructing local DRSDs using Bodhisattva’s face model. In Fig. 2(a), for a given vertex in 3D meshes (such as Point 1), the points in its vicinity are selected by a sphere. Then, a planar circle is used as boundary condition to implement Harmonic Maps, obtaining the sub-meshes in parametric plane (Fig. 2(b)). By adding curvature to the vertices surrounding the given vertex, the local DRSD (Fig. 2(e)) is constructed. Note that the local DRSDs have been enlarged for the purpose of visual effect. The beauty of using Harmonic Maps for local DRSD is that the spring constants used only depend on the angles or length in meshes. Once the triangles are given, the spring constants are easily obtained by the relationships between points and its neighborhoods. In our method, Floater’s mean-value coordinates [36] are used to implement Harmonic Maps:

$$k_{ij} = \left( \frac{\tan \varphi_1 + \tan \varphi_2}{2} \right) \|l_{ij}\|$$ (6)

where the angles $\varphi_1$ and $\varphi_2$ and distance $l_{ij}$ are shown in Fig. 3. Now the local DRSD for surface patches is obtained, and the surface’s intrinsic attributes are inherited in planar meshes.

The main difference between our local DRSD and Zhang’s Harmonic Shape Images is that different spring constants are used to describe surface properties. When there are large obtuse angles from Gaussian noise or low resolution, Eck’s spring
constants $k_{ij} = \cot \alpha_1 + \cot \alpha_2$ (Fig. 2) used in Zhang's method will give negative value, thus violating the bijective property that any mesh parameterization technique should have. In this situation, the Harmonic Shape Images in [27] are not suitable to represent the surface patches. However, in our method, this problem can be avoided. Unlike Eck's methods, Floater's spring constant $k_{ij}$ is always positive. When mapping the local mesh surfaces to circular domain, it is important to avoid singular and guarantee the bijectivity for discrete Harmonic Maps. Therefore, by local DRSD, the surface properties can be described and inherited in low-dimensional space.

A good 3D shape description should be view-independent, preservative to surface properties and easy to construct. With these traits, our local DRSDs are applicable to represent free-form surfaces. In modified Spin-Image [33], a local spherical coordinate system was defined at vertex $s$, which was regarded as the origin. Then, a tangent plane passing through the origin was established, so as to calculate the average distance from all the points in the vicinity of $s$ to the plane. Since defining the tangent plane involves the normal calculation of origin, principal component analysis (PCA) was carried out in spherical domain, and the second important principal axis was used as the normal vector. In local DRSD, all the assistant implementations towards tangent plane are needless. Based on the principle of minimal Dirichlet energy, the 3D triangles in local meshes are harmoniously stretched on the plane, where linear solver is used to implement parameterization. Then, all the points in local meshes are indexed and accumulated as the invariant to rigid transformation. In addition, our local DRSDs are robust to sampling resolution. If there is a point $s_i$ in one triangle shown in Fig. 3, linear interpolation technique is used to calculate the curvature at this point, according to the known curvature value of three vertices $v_i, v_m, v_j$ on mesh surfaces.

### 3.2. Generating the candidates

When the local DRSD for selected vertex on sensed surface are obtained, the next task is to calculate the corresponding point of the vertex on model surface. First, local DRSDs with respect to the vertices with high curvature on model surface are constructed to establish a standard DRSD library in advance. Since any planar point in the circles (Fig. 2(b)) can be described by an angle and a radius, the curvature value in local DRSD (Fig. 2(e)) is discretized by polar coordinates. Assume that $N$ points in polar coordinates are sampled. Then, the normalized correlation coefficient $NCC_{PFO}$ introduced in [37] can be used to quantify the similarity between selected vertex and its potential correspondence in the library

$$NCC_{S,M} = \frac{N \sum p_i q_j - \sum p_i \sum q_j}{\sqrt{(N \sum p_i^2 - (\sum p_i)^2)(N \sum q_j^2 - (\sum q_j)^2)}}$$  \hspace{1cm} (7)$$

where $S$ and $M$ denote the sensed surface and the model surface, respectively. Symbols $p_i$ and $q_j$ denote the curvature of sampling points by linear interpolation (refer to Fig. 3), from sensed surface and model surface, respectively. From the generation process of local DRSD, it can be seen that there may be a rotation difference between local DRSD of seed and the local DRSD of its correspondence. This means that normalized correlation coefficient is a function of rotation angle $\phi$. In order to get rid of the rotation dependence, the Pseudo-Polar Fourier Transform (PPFT) in [38] is
employed to obtain the maximal correlation coefficient:

\[
NCC_{S,M} = \max_{\theta} NCC(\theta)
\]  
(8)

By comparing the NCC of local DRSDs, pair of corresponding points can be found. We refer to these pairs of point correspondences as candidates. Let the list of the obtained candidate vertices be denoted by \(L = \{V_1, V_2, \ldots, V_m\}\). The candidates include two kinds of points: one is the seed point \(s_i\) on \(S\) and the other is its corresponding point \(m_i\) on \(M\). In our experiments, \(m\) seeds (not more than 80) are selected to generate the candidates. It must be pointed out that the seeds are not selected randomly. It observes that many pseudo-correspondences appear when the vertices are chosen in regular regions. Thus, we restrict ourselves to select the seeds in highly curved areas. With distinguishing local DRSDs, false correspondences on \(M\) would be severely reduced.

3.3. Rejecting outliers for transformation

From the candidates in Section 3.2, we attempt to select several triplets to compute the rigid transformation. These triplets are obtained by randomly selecting three seeds in \(L\) each time, and we get a group of triplets as follows:

\[
T = \{(S^1,M^1),(S^2,M^2),\ldots,(S^m,M^m)\}
\]  
(9)

where \(S^i = \{s^i_1, s^i_2, s^i_3\}\) denotes the set of three seeds in triplet, and \(M^i\) denotes the set of corresponding points with respect to \(\{s^i_1, s^i_2, s^i_3\}\). If the candidates are accurate, one-to-one in two surfaces, the transformation parameters can be uniquely computed. However, due to multiple correspondences for each seed, there may be outliers on \(M\). Before computing the rigid transformation, the outliers should be identified and removed. As shown in Fig. 4, four types of selective seeds may occur:

Type (a): The seed is in the boundary of sensed surface but it finds its correspondence on \(M\).
Type (b): The seed is in non-overlapping regions but it finds false correspondences on \(M\).
Type (c): The seed is in overlapping regions but it finds multi-correspondences on \(M\).
Type (d): The seed is in overlapping regions and it finds only one correspondence on \(M\).

In Type (a), for the vertices in the boundary of sensed surface, the curvature information from discrete data is not accurate. The seed is not suitable to participate in the calculation of transformation parameters. By checking whether the seed belongs to the boundary, this kind of error can be avoided. Of course, it is an easy case with little computing time.

In Type (b), the candidates provide false corresponding points. In the application of reverse engineering and face recognition, the given 2.5D range images are captured by the laser sensor from different views, and the non-overlapping data is inevitable. When these outliers are used to compute the transformation, the registration error will be large. For eliminating the outliers, one needs detecting the overlapping regions in 3D space. So far, it is still a challenging task in 3D shape registration.

In Type (c), it is the most common case encountered in shape registration. Some reasons account for this result. Since the local DRSD describe the surface properties around the seed, multiple correspondences are usually unavoidable. Furthermore, the existing noise and sampling error in measurement also lead to this problem. When the surfaces comprise of a large number of similar regions with low curvature, this problem is particularly obvious. Johnson and Hebert [12] employed geometric consistency to remove outliers. This algorithm is simple to implement. However, for the objects with similar surface patches, there will be a large number of multi-correspondences, making the computation of consistency measure very time-consuming. In Type (d), it supplies the ideal case. If all the triplets belong to Type (d), the transformation is easily computed. Unfortunately, the occurrence of such situation likely exists, only when the seeds lie in high-curvature areas. It can be seen that the approach to selecting the seeds with high curvature in Section 3.2 brings the probability of ideal triplets to a certain extent.

To deal with Type (b) and Type (c), we adopt a new distance metrics, distance root mean squared error (dRMS), originally used in computational molecular biology for comparing the protein structure [39]:

\[
dRMS(S,M) = \sqrt{\frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left(\frac{1}{|s_i-s_j|} - |m_1-m_2|\right)^2}
\]  
(10)

where \(N\) denotes the number of corresponding point matches. In this paper, first an initial triplet, satisfying the condition in (10), is randomly selected from \(T\). Then, the coordinates of other triplets are added in (10), one by one, in order to verify whether the remaining triplets are valid. Here an important task is how to define the threshold of \(dRMS\) to determine reasonable triplets. Gelfand et al. [5] have proved that the upper bound of \(dRMS\) was \(\sqrt{2}\) times coordinates mean squared error (cRMS):

\[
cRMS(S,M) = \frac{1}{N} \sum_{i=1}^{N} |R_s + t - m_i|
\]

where \(R\) and \(t\) denote the rotation matrix and translation vector, respectively. For correct registration, we assume that cRMS should not exceed \(\delta\), where \(\delta\) denotes the sampling resolution. Therefore, in our experiments, the accepted value of \(dRMS\) is set as \(\sqrt{2}\delta\). By this way, valid triplets in \(T\) are extracted one by one, and we obtain an updated group of triplets \(T = \{(S^1,M^1),(S^2,M^2),\ldots,(S^m,M^m)\}\). By a simple judgment from (10), the seeds belonging to Type (b) are rejected, and false correspondences are removed for the seeds belonging to Type (c). It should be pointed out that after applying the \(dRMS\), there still may be potential multi-correspondences, which meets the constraint of (10). In our experiments, only the triplet with minimal \(dRMS\) is added to the updated triplets \(T\). We treat these triplets as valid triplets. After that, all the seeds and their correspondences are one-to-one. Note that when the three seeds in one triplet are adjacent in meshes, the transformation from the triplet would cause a great mismatch error. For avoiding this problem, the shortest edge of triangles \(l_{\text{min}}\)

Fig. 4. Position of the selective seeds in surfaces: for seed a, it is in the boundary of sensed surface; for seed b, it is in non-overlapping regions of two surfaces and for seeds c and d, they are both in overlapping regions.
in each triplet should be set, and experimental results show that it can be selected between \( l_{\text{min}} = (3 - 10) \delta \).

The main advantage of using \( dRMS \) is that it is simple to calculate and is a stable invariant to rigid transformation. Traditionally, the \( cRMS \) is used as the measure of the residual error in registration. However, before the registration error is calculated, the transformation parameters must be obtained. If the computed rigid transformation is inaccurate, it bears the risk of useless calculation. For this problem, Liu [9] pointed out that new methods should be developed to predict the registration quality before the registration (or matching) really takes place. As the invariant to rigid transformation parameters, \( dRMS \) with surface intrinsic properties just fit this requirement. Depending on the threshold \( \sqrt{2} \delta \), reasonable seeds are not missed and false correspondences are not accepted. Only valid triplets are preserved to solve the best transformation, including rotation vectors \( R_t \) and translation vector \( t_t \).

3.4. Refining rigid transformation

Because of the existence of sampling error and noise, the obtained \( R_t \) and \( t_t \) may not be the optimal transformation parameters. However, they provide a coarse initialization to iterative methods such as ICP for refining the rigid transformation. One important requirement of ICP algorithm [7] is that the points used must have correct correspondences on model surface. It means that the points on sensed surface should be in overlapping regions. Unfortunately, the overlapping regions for two surfaces are still unknown to us. Based on the surface consistency test, exhaustive exploration of 3D space can be used to find the close points in 3D space as little as possible.

Thanks to the valid triplets \( T \), we can implement a variational ICP algorithm using a different sampling strategy. In \( T \), all the seeds are considered as valid points in overlapping regions. For the seeds in a triplet, there are several points inside the selected sphere. We define these points as \( B' = \{ B_1', B_2', B_3' \} = \{ \{ s'_1, s'_2, \ldots, s'_N \}, \{ s''_1, s''_2, \ldots, s''_N \}, \{ s'''_1, s'''_2, \ldots, s'''_N \} \} \). Points in \( B' \) are regarded as the potential points in overlapping regions. Applying the obtained transformation \( g_o = (R_o, t_o) \), we get \( s_i = R_s s_i + t_t \) where \( s_i \) belongs to \( H = \{ B', B'', \ldots, B''' \} \). If the error \( |s_i - m_i| \leq d_{\text{min}} \) where \( d_{\text{min}} \) is a given threshold (here \( d_{\text{min}} \) is set as \( \delta/2 \) since the influence of noise and resolution is little), the corresponding point matches \( (s_i, m_i) \) are reserved as the valid data for fine registration. Otherwise, the point \( s_i \) is removed. After this process, we obtain a list of point matches \( P = \{ V_1, V_2, \ldots, V_m \} \), \( V_i = (s_i, m_i) \), and employ them to implement iterative registration via standard ICP algorithm. At this stage, the optimal rigid transformation \( g_o = (R_o, t_o) \) is obtained, from which the sensed surface is registered to the model surface.

4. Evaluate the registration accuracy by global DRSD

After the shape registration is achieved in Section 3, we proceed with the computation of the registration error to determine whether the obtained rigid transformation best aligns the two surfaces. For the objects with partially overlapping data, it is an interesting and challenging task to evaluate the registration accuracy. In this paper, by constructing the global DRSD, we reduce the 3D problem of assessing shape registration to a 2D matching problem.

4.1. Constructing global DRSD

In this section, ABF++ is used to construct global DRSD, describing overall shape of the underlying surface in plane. In global DRSD, angle-preserving parameterization method ABF++ is implemented to map 3D mesh surface onto a 2D domain in continuous manner. The surface intrinsic attributes, mean curvature, are assigned to the points on planar meshes. As an example, Fig. 5 shows the process to construct DRSD using the Buddha model. After ABF++ parameterization, the planar meshes (Fig. 5(b)) are generated from original 3D meshes (Fig. 5(a)). When the curvature information is added to the planar meshes, the global DRSD (Fig. 5(c)), which represents the overall shape of surface in plane, is obtained.

Shape deformation is usually unavoidable when implementing ABF++ parameterization to stretch the surfaces onto plane in global DRSD. The obtained domain triangles will be slightly different from the shape of the original triangles, leading to angle and area distortion. According to Gauss’s theory [41], only the developable surfaces with vanishing Gaussian curvature \( K(p) = 0 \) at all points on surface can provide the global DRSD without any distortion. This kind of surfaces includes planes, cones, cylinders and stretching surfaces. For general free-form surfaces, the preservation of 3D shapes in global DRSD is the major concern. This issue has also paid close attention to other geometric processing applications such as remeshing and texture mapping. Fortunately, due to the characteristics of preserving the geometric shape, global DRSD supplies a choice for us to represent overall 3D shapes with minimal distortion. As demonstrated in Sheffer’s works, angle-based flattening methods introduce significantly less stretch into the parameterization in the regions of surfaces that have high Gaussian curvature. This means that in global DRSD the angle relationships, between one triangle and its adjacent triangles in different spaces, are kept unchanged as
much as possible regardless of the curvature features. Therefore, we can naturally represent the surfaces in low-dimensionality space by global DRSD.

An important point, which governs our evaluation framework for registration accuracy, is that global DRSD can establish the mapping relationships for the points in different Euclidean spaces, and conserve the continuity properties of surfaces in plane. Thus, the complex geometric processing problem, calculating the registration error for partially overlapping data, can be performed in 2D space. In addition, global DRSD has simple calculation; it preserves both the shape and continuity of mesh surfaces; it is one-to-one and intrinsic to underlying surfaces. All these advantages motivate us to apply the global DRSD in evaluating registration accuracy. The details are provided in the following section.

### 4.2. Calculating registration error

When sensed surface and model surface are integrated into a common coordinate system, global DRSDs are constructed to compare the similarity in 2D plane, where a chessboard is used to verify the overlapping regions. The steps using global DRSDs to calculate the similarity are (a) selecting three seeds \( s_1, s_2, s_3 \) and finding their correspondences \( \{t_1, t_2, t_3\} \), (b) building a plane chessboard \( \pi \) from the coordinates of points in global DRSDs and (c) verifying the overlapping regions \( \tau \) to compare the similarity. The example is illustrated in Fig. 6.

In step (a), to build a plane chessboard, three seeds \( S = \{s_1, s_2, s_3\} \) in arbitrary triplet of \( T \) are chosen. As shown in Fig. 6(a), for each seed \( s_j, j = 1, 2, 3 \), its renewed point after optimal transformation \( g_0 = (R_0, t_0) \) is defined as \( s'_j = g_0 s_j \). Note that points \( m_{ij}, j = 1, 2, 3 \) can be regarded as the closest point of \( s'_j \) on model surface. Then, \( s'_j \) is projected onto the triangles \( A_k \), \( k = 1, 2, ..., r \) with 1-ring of \( m_{ij} \), and we obtain some projecting points \( t_k, k = 1, 2, ..., r \). If \( \min|s'_j - t_k| \leq |s'_j - m_{ij}|, \) \( t \in \{t_1, t_2, ..., t_r\} \), let us define \( t = t_k \). Otherwise, \( t = t_k \). After this, we obtain a fixed triplet \( T = (S, M) \) where \( M' = \{t_1, t_2, ..., t_r\} \).

In step (b), to compare the similarity, the non-collinear seeds \( \{s'_1, s'_2, s'_3\} \) in step (a) are used to fix the positions that two global DRSDs share. As shown in Fig. 6(b), the global DRSD with respect to sensed surface is constructed in plane. In terms of the mapping relationship, the coordinates \( C = (c_1, c_2, c_3), (c_2, c_3, c_1), (c_3, c_1, c_2) \) with respect to seeds \( \{s'_1, s'_2, s'_3\} \) can be obtained from the global DRSD. Then, when implementing the ABF++ to construct the global DRSD of \( M \), the plane coordinates of points \( t'_k, k = 1, 2, 3 \) are imposed to pass through the same positions \( C \). Based on the extreme value of \( x, y \) coordinate from two global DRSDs, a plane chessboard \( \pi \) composed of \( m_{cx}, m_{cy} \) cells is built. The numbers \( m_{cx}, m_{cy} \) depend on the resolution:

\[
m_{cx} = n_c \Delta c_x / \delta, \quad m_{cy} = n_c \Delta c_y / \delta
\]

where \( n_c \) denotes a constant, and \( \Delta c_x, \Delta c_y \) denotes the maximal difference value of \( x, y \) coordinates for two global DRSDs. With the built chessboard, we will detect the overlapping regions for registration accuracy.

In step (c), for each crossing point \( c'_l, l = 1, 2, ..., m_{cx} \), \( n = 1, 2, ..., m_{cy} \) in chessboard we will check whether the point belongs to a triangle on sensed surface, as well as belongs to another triangle on model surface. In this case, point is identified as the points in overlapping regions. By the linear interpolation (Fig. 3), the curvature value \( p_{il} \) for point \( c'_l \) is calculated from the points on \( S \). In the same way, another curvature value \( q_{il} \) is calculated from the points on \( M \). By the strategy of checking the crossing points, overlapping regions \( \tau \) can be verified. Therefore, with mean curvature \( p_{il} \) and \( q_{il} \), the registration error is computed by the normalized correlation coefficient in (7), and then compared to a given threshold \( \text{NCC}_{\text{G-min}} \), we can determine whether the obtained optimal transformation is satisfactory.

The main advantage of using global DRSDs to evaluate the registration accuracy is that the calculation of overlapping regions and registration error can be performed in plane. The exhaustive-search algorithms in 3D space, aiming to verify the overlapping data for two objects, are not required. In global DRSDs, ABF++ can map 3D meshes to 2D meshes with shape-preserving properties. Then a chessboard is used to identify the overlapping data. For correct alignment, the images have similar curvature information in overlapping regions, and it will output high global normalized correlation coefficient.

### 5. Analysis and discussion about hybrid DRSD algorithm

#### 5.1. Time complexity

First, we analyze the time complexity of our hybrid DRSD algorithm. For almost all registration algorithms using surface representation, the search for closest points is required to obtain a spherical or cylinder regions centered at a seed. In the following, we will only focus on the time complexity of other steps. For clear description, we define the numbers of points on \( S \) and \( M \) as \( n_S \) and \( n_M \), and the numbers of corresponding triangles as \( n_{T_S} \) and \( n_{T_M} \) respectively. For Sections 3 and 4, we explain the time complexity.

Section 3 mainly consists of four steps: (a) select \( m \) seeds to construct local DRSDs, (b) compare the local NCC between seeds and their potential correspondences, generating the candidates, (c) reject the outliers in a group of triplets from the candidates,
and compute the initial rigid transformation and (d) refine the transformation parameters using variational ICP.

In step (a), m vertices with high-curvature value on S is selected as the seeds. When the curvature for each vertex is known, it needs a complexity of \(O(m)\). In step (b), a standard library is established. We define that the number of local DRSDs in the library is \(n_l\) satisfying \(n_l < n_M\). Then, for each seed \(s_i\) on S, we perform \(n_l\) comparison; thus the complexity is \(O(mn_l)\). It can be seen that the total time complexity of steps (a) and (b) is \(O(mn_l)\).

In step (c), for all the triplets in \(T\), the calculation of \(dRM\) is performed to remove the false correspondences. Assume that for each seed there is an average of \(g\) correspondences. Then, for each triplet, we must test \(g^3\) candidate correspondences on \(M\), which can be seen in Section 3.3. The complexity is therefore \(O(mg^3)\). In addition, another test is implemented to check whether the triangles in each triplet meet a minimal edge-length requirement. Assume that for each seed there is an average of \(g\) correspondences after the \(dRM\) test. It results in a complexity of \(O(mgn)\).

Since \(m \leq m\) and \(g \leq g\), the total complexity of step (c) is \(O(mg^n)\).

In step (d), since the time complexity of searching closest points is not considered, the computing time only depends on the error comparison. Assume that for each valid seed there is an average of \(h\) points in its defined sphere. The complexity of step (d) is \(O(mh)\). Therefore, the total complexity of steps (a)-(d) is \(O(mn_l) + O(mg^3) + O(mh)\). Since numbers \(m\) and \(M\) are in the same level, the time complexity can be regarded as \(O(n_l) + O(g^3) + O(h)\). Note that both \(n_l\) and \(h\) are usually less than \(g^3\). In summary, the overall complexity of calculating the optimal transformation is \(O(g^3)\). It means that the time complexity mainly depends on the number of selected seeds and the number of their correspondences on \(M\). Now we compare the time complexity of our algorithm with Spin-Image. Spin-Image requires a complexity of \(O(g^3n^2_M)\), where \(n_M\) denotes the number of points represented by 2D image. It is clear that our algorithm is faster than Spin-Image if \(mg < n_M^2\). In practice, there exists \(mg \leq n_M\) and \(n_M \geq n^3_M\) hardly occurs.

Section 4 includes three steps that have been shown in Section 4.2. In step (a), for each seed, it requires to calculate the points with 1-ring of seed’s correspondence \(m^e\) result in a complexity of \(O(n_M^2)\). In step (b), for both sensed surface and model surface, global DRSDs are implemented. This process needs a complexity of \(O(k_1)\), where \(k_1 = \max(n_M, n_S)\). In step (c), for each crossing point in chessboard, it needs to do a search for triangles on S and \(M\). This needs a time complexity of \(O(k_2)\) where \(k_2 = \max(n_M, n_S)\). It can be seen that the overall complexity of Section 4 is \(O(n_M^2) + O(k_1) + O(k_2)\). Note that the number of vertices and the number of triangles are in the same level. In Section 4, the total complexity is \(O(k_1)\). Since \(\max(n_M, n_S) \geq n^3_M\) hardly occurs, the time complexity of Section 4 is also less than \(O(g^3n^2_M)\).

5.2. Limitations

The hybrid DRSD registration algorithm includes two processes: implementing local DRSD to compute the transformation parameters and implementing global DRSD to evaluate the registration accuracy. The main limitation within the algorithm is that when we carry out the global DRSD in the second process, the mesh surfaces should be a consecutive object with only one boundary. Fig. 7 gives two failed examples when constructing global DRSD. In these two examples, ABP+ cannot be stably performed.

The shape registration is also limited to the quality of input data. For example, if the sampling resolution is low, the mean curvature used in both local DRSD and global DRSD is not accurate. It will affect the results of computing transformation parameters and evaluating registration accuracy. Therefore, before implementing registration, some preprocesses such as noise reduction, filling internal holes and remeshing are required to ensure high-quality meshes. Nowadays, it is an easy case to handle such problems using commercial software such as Geomagic Studio. It should be pointed out that our algorithm is limited to symmetry features, which is also a common problem in most of other registration methods using surface representation. Although clustering methods such as k-means algorithm or ISODATA algorithm can solve this problem, it leads to large time consumption. In addition, the area of the overlapping regions should be greater than 50% of the small surface. This consists with other surface representation-based registration methods [22,27].

6. Experimental results and performance analysis

In this section, both real and synthetic images are used to test the hybrid DRSD registration algorithm. First, several experiments are implemented to compare our algorithm with Spin-Image from the viewpoint of registration accuracy and efficiency. Then, compared with Zhang’s algorithm, the robustness of local DRSDs is tested. In addition, to observe the robustness of global DRSD, a large number of experiments are implemented to test the proposed algorithm, using real images with varying resolution, fixed points, noise or occlusion. Salvi et al. [25] have given a specific analysis between LSCM and Harmonic Maps, showing that LSCM is more suitable for global surface representation than Harmonic Maps. In this paper, we focus on comparing our global DRSD with LSCM algorithm.

Fig. 7. Two examples of failure in implementing global DRSD: (a) the image has more than one part and (b) the image has two boundaries.
6.1. Registration accuracy and efficiency

In this section, three examples (Shell, Warrior and Blade shown in Fig. 8(a, b), (e, f) and (i–k)) are used to compare our algorithm with Spin-Image. All the algorithms are programmed using Matlab 7.0 and operated in a 2.66 GHz Inter processor with 3GB RAM. For Shell model, there are 3370 and 3565 points on sensed surface and model surface, respectively. The numbers of corresponding triangles are 6538 and 6955, respectively. For the Warrior (Blade) model, the numbers of points are 5255 (2180) and 5810 (2222) on sensed surface and model surface, containing 9979 (4143) and 11,202 (4218) triangles, respectively. The average distances of vertices about two examples are 2 mm, and the proportion of overlapping regions is about 55%. In these experiments, the number of seeds, the minimal edge length and the threshold of iterative registration in Section 3 are set as $m = 50$, $l_{\text{min}} = 5\delta$, $d_{\text{min}} = 0.5\delta$. Moreover, the constant number in Section 4 is set as $n_c = 3$.

The registration results are shown in Fig. 8 and Table 1. It can be seen that before alignment, the positions of sensed surface and

![Fig. 8. Registration results of the Shell, Warrior and Blade models: (a, b, e, f, i, j) partial images before registration; (c, g, j) registered image using local DRSD without iteration and (d, h, k) well-registered image after iteration.](image)

<table>
<thead>
<tr>
<th>Examples</th>
<th>Algorithms</th>
<th>Number of candidates Before rejection of outliers</th>
<th>Number of valid triplets</th>
<th>Global NCC Before iteration</th>
<th>Consuming time Before iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shell</td>
<td>DRSD</td>
<td>1249</td>
<td>69</td>
<td>0.9437</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Spin-Image</td>
<td>1837</td>
<td>201</td>
<td>0.9349</td>
<td>81</td>
</tr>
<tr>
<td>Warrior</td>
<td>DRSD</td>
<td>2853</td>
<td>102</td>
<td>0.9227</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>Spin-Image</td>
<td>3259</td>
<td>516</td>
<td>0.9253</td>
<td>708</td>
</tr>
<tr>
<td>Blade</td>
<td>DRSD</td>
<td>931</td>
<td>52</td>
<td>0.9520</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Spin-Image</td>
<td>1205</td>
<td>164</td>
<td>0.9483</td>
<td>68</td>
</tr>
</tbody>
</table>
model surface (Fig. 8(a, b), (e, f), (i–k)) show considerable difference in 3D space. For the Shell model, 1249 candidates are initially calculated by constructing local DRSDs. The outliers from the candidates are filtered out using the constraints of distance root means squared error as well as minimal edge length, and only 69 corresponding point matches are reserved. From the correct correspondences, 8 valid triplets are adopted to compute rigid transformation parameters, registering the sensed surface to the model surface (shown in Fig. 8(c)). Then, the proposed variational ICP in Section 3.4 is applied to further refine the rigid transformation. The final registration result is shown in Fig. 8(d). It can be seen that two partially overlapping surfaces are correctly aligned.

After that, global DRSDs for Shell models are constructed to evaluate the registration accuracy. In Table 1, the calculated global NCC is 0.9581, which is close to the registration error of Spin-Image. It shows that the registration result of our algorithm is comparable with that of Spin-Image.

When the hybrid DRSD algorithm is used for the Warrior model with more complex features than the Shell model, we still obtain a satisfactory registration. Two reasons account for these results. One is the discriminate similarity metric from the local DRSD. Our local DRSD makes full use of the points surrounding the seed to attribute the local surface patches, and robust NCC is adopted to seek correspondences for candidates, where false point matches are refused by the rejection strategy in Section 3.3. The other is the application of variational ICP algorithm. In fact, only few reliable points distributed on sensed surface are used to implement iterative registration, which is advantageous to registration accuracy and efficiency.

The time consumption can be seen in Table 1. In these two experiments, the speed of our algorithm is faster than that of Spin-Image. Note that when the number of candidate increases from 1837 to 3259, the time complexity of Spin-Image expands quickly. The reason is that when implementing the geometric consistency, the time consumption of Spin-Image depends on the number of candidate point correspondences. If $m_{3D}$ points are used to calculate Spin-Image for candidate corresponding points, the time complexity is $O(g^2 m_{3D})$. Large number $m_{3D}$ leads to square-increasing time consumption. However, the time complexity of our algorithm is mainly related to a small quantity of seeds, as well as limited number of corresponding points. In these experiments, the hybrid DRSD algorithm receives fast registration, which is consistent with the analysis in Section 5.1.

6.2. Robustness analysis of local DRSD

First, using Planck’s Face model (http://shapes.aimatshape.net/viewmodels.php), we test the local DRSD for seeking correspondences. The mean distance of the vertices in meshes is increasing time consumption. However, the time complexity of our algorithm is mainly related to a small quantity of seeds, as well as limited number of corresponding points. In these experiments, the hybrid DRSD algorithm receives fast registration, which is consistent with the analysis in Section 5.1.
2 mm. The sensed surface contains 3251 points and 5972 triangles, and the model surface contains 4486 points and 7736 triangles. In Fig. 9, three seeds $X = \{p_1, p_2, p_3\}$ (Fig. 9(a)) on $S$ are selected to calculate the correspondences $Q = \{q_1, q_2, q_3\}$ on $M$. With different spring constants, the maximal normalized correlation coefficients for point matches are given in Table 2, where both mean curvature and Gaussian curvature are used. It shows that when finding correct correspondences, Floater’s mean-value coordinates are competent to implement Harmonic Maps with high local NCCs. Note that at point 2 the NCCs using Eck’s spring constants are much smaller than our method (0.9528, 0.9452 vs. 0.9802, 0.9892). The reason is that existing obtuse angles cause “triangles flip”. It implies that it may be unstable for Zhang’s method to perform local representation for correspondences when there are obtuse angles.

Then, employing hexagon or square polygons as boundary conditions, another experiment is implemented to test the local DRSD. The calculated point matches using circular boundary are defined as known feature points, and other boundary polygons are input to calculate the local NCCs at point 2. The obtained local DRSDs and compared results are given in Fig. 9(e–h) and Table 3, respectively. In Table 3, the calculated local NCCs using other boundary conditions are smaller than 0.9600. It can be seen that circular boundary is more suitable to construct local DRSD than both hexagon and square boundaries.

6.3. Robustness analysis of global DRSD

6.3.1. Test on sampling resolution

First, using the Shell and Face models, we test the robustness of global DRSD to sampling resolution. The initial sampling space for two examples is 1 mm. The other test data (Fig. 10(a–c)) with space from 1.5 to 5.5 mm is obtained by rarefying the original points. The numbers of points change from 10,617 to 955 for the Shell model, and from 11,989 to 1102 for the Face model. By matching the variational surfaces to the original surfaces, the global normalized correlation coefficients are calculated. The registration result is given in Fig. 11. When the resolution is gradually dropping to only 25% of original resolution (sampling space 4 mm), NCCs of our algorithm still remain stable in the range of $[0.90, 1.0]$. In contrast, as the sampling space increases, there is a relative fast reduction of NCCs for LSCM, especially for the Face models with high-curvature features. It shows that our
method is robust to resolution with high NCCs when achieving correct alignment. For 3D meshes with poor resolution, global DRSD supplies more reasonable planar meshes than LSCM. The reason is that ABF++ generates minimal deformation during parameterization from 3D space to 2D space.

Moreover, when the sampling space exceeds 4 mm, low NCCs for both the algorithms are obtained. This is because the meshes are too sparse to obtain accurate curvature information, leading to not credible surface properties in global DRSD.

6.3.2. Test on fixed point

When implementing global surface representation, both ABF++ and LSCM need some fixed points in plane. In this experiment, we test the robustness of our algorithm to different positions of the fixed points. Fig. 12 shows the models with 1 mm sampling space, from which the sensed surfaces are generated with low resolution (3.5 mm). Points 1 and 2 are two fixed points, the third point is set in potential positions from point 3 to point 10. Varying the position of the third point, the obtained NCCs are given in Fig. 13. Our algorithm almost gives stable normalized correlation coefficients, but LSCM algorithm generates shake in different positions. It means that we can construct global DRSDs by fixing three non-collinear seeds regardless of their positions. However, LSCM algorithm requires choosing optimal positions to get minimal registration error for correct registration, which is usually difficult to carry out in practice.

The robustness of global DRSD to the fixed positions derives from the characteristic of ABF++ parameterization. ABF++ avoids some anomalies of conventional parameterization methods. For example, it applies free boundary conditions; so all the triangles in mesh can be flatted onto parametric plane without boundary restrictions. Moreover, a set of constraints on interval angles and Newton-based optimization method are used to perform parameterization. It is helpful to relieve the relative distortions during parameterization. Experimental results [32] show that ABF++ brings an order of magnitude less stretch than linear parameterization method LSCM, especially for the regions with high-curvature features. Therefore, the surface properties can be kept consistent in different spaces. Fixing three non-linear points in parametric plane corresponds to the situation of fixing three corresponding points in 3D space. For correct alignment, the obtained NCCs with respect to different fixed positions are analogous.

6.3.3. Test on noisy data

Next, we test our algorithm under the Gaussian noise using both the Shell and Warrior models. The Gaussian noise $N(0,\sigma^2)$ is added to each point of the surfaces, where the unbiased variance $\sigma$ is concerned with the resolution $\delta; \sigma = \delta \times r\%$, $r \in [0, 0.25]$. The surfaces with different Gaussian noises and the registration

![Fig. 12. Test positions of the third fixed points for the Shell and Face models.](image)

![Fig. 13. Registration results of DRSD and LSCM algorithms under different fixed points. The deviations of global NCCs for two models are 0.0013 vs. 0.0265 and 0.0051 vs. 0.0241, respectively.](image)
results are shown in Figs. 14 and 15, respectively. In presence of varying amount of noise, our global DRSDs always keep high normalized correlation coefficients for correct alignment. Note that for the Warrior model the NCCs of LSCM algorithm are really poor when the unbiased variance of noise increases to 20% resolution.

The data show that our algorithm appears to be more robust to the Gaussian noise than LSCM. For the Warrior model, there are a large number of high-curvature regions, such as pieces of armor, hair lace and ornament in the neck (bottom row of Fig. 14). When the meshes are stretched onto the plane, linear method LSCM will generate large distortion, especially when the surfaces are excessively polluted by the Gaussian noise. However, non-linear method ABF+ naturally minimizes distortions as much as possible, and can supply relatively accurate results. Therefore, high NCCs for correct alignment is expectable.

Fig. 14. Noisy data of the Shell (top row) and Warrior (bottom row) models under zero-mean additive Gaussian noise with different unbiased variance \( \sigma \): (a) \( \sigma = 0 \); (b) \( \sigma = 5\% \delta \); (c) \( \sigma = 15\% \delta \) and (d) \( \sigma = 25\% \delta \).

Fig. 15. Registration results of DRSD and LSCM algorithms under Gaussian noise.
6.3.4. Test on occlusion

In the final experiment, the purpose is to demonstrate the robustness of global DRSD to occlusion. The examples used are shown in Fig. 16(a), and their variations (Fig. 16(b–d)) are obtained by cutting partial meshes in different positions. The registration results are given in Table 4. It can be seen that the normalized correlation coefficients of global DRSD are basically stable, in spite of the cutting parts of the objects. The fine property of preserving shape of ABF++ accounts for this result. However, for LSCM registration, it seems prone to be affected by the occlusion, especially for the models with high-curvature features (Warrior model).

In addition, another experiment under different percentages of overlapping regions is also implemented to further test the robustness of global DRSD. The test data and the constructed global DRSDs are shown in Fig. 17. The normalized correlation coefficients are given in Fig. 18. From the results, it can be seen that NCCs remain more than 0.9000, in spite of the severe

![Image of Fig. 16: Global DRSDs for test models and their variations under occlusion in different positions.](image)

![Image of Table 4: Calculated normalized correlation coefficients between original global DRSDs and their variations for the Shell and Warrior models.](image)
occlusion when the percentages reach up to 50%. This attributes to fine shape-preserving property of ABF++. In our experiments, 50% overlapping coefficient is enough to supply highly normalized correlation coefficient for correct alignment.

7. Conclusions

In this paper, we present a novel 3D shape registration algorithm for partially overlapping data using hybrid DRSD. Our approach unifies both local parameterization and global parameterization techniques to represent the 3D meshes of free-form surface in planar domain. Hence, searching correspondences, detecting overlapping regions and calculating registration error all can be carried out in low-dimensional space. This leads to a fast and accurate registration approach, which has discriminating similarity metric to obtain correspondences for rigid transformation and simple calculation of registration error in overlapping regions. We also illustrated our approach on datasets of real and synthetic images, and gave comparison to the similar approaches such as Spin-Image and LSCM presented in this paper. Our algorithm is accurate and robust to different resolution, noise and missing data, which usually exist in practical measurement. Meanwhile, our hybrid DRSD algorithm can be strengthened by ensuring high-quality input of 3D meshes. For instance, the noisy data causes false correspondences for selected seeds, resulting in inaccurate transformation parameters and poor registration accuracy. Therefore, before implementing shape registration, preprocessing such as filtering noise, filling holes and remeshing are greatly appreciated.

Furthermore, our method has sound mathematical meanings, and it can also serve as a ubiquitous tool for a wider range of applications.
applications such as surface localization, toolpath generation, face recognition, remote sensing and robot navigation. In the future, we would like to apply hybrid DRSD algorithm in these more challenging research topics.

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References


